USE OF MATHEMATICS IN ETHICS

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Abstract

In this article, we review usage of basic Math operators like Set Theory, Probability Theory in Ethics by Jeremy Bentham [1]. Further, we present ideas how Math thinking & reasoning can support ethical values in society.

Introduction

Ethical reasoning firstly requires consideration of values which can be viewed as follows:

1) Numbers (Positive, Negative or Fractional)
2) Standard Mathematical Operators (addition, subtraction, multiplication, division)
3) Math Theories

According to Jeremy Bentham, Math is needed in Ethical Reasoning for manipulation of values and producing an answer to guide our actions. He defined a Scale for representation of things like pain and pleasure and applied Calculus over factors to evaluate interest, based on the following:

- Intensity
- Duration
- Extent
- Certainty

Intensity & Duration

Jeremy Bentham gave the following formula as under:
Value of Interest = f (Intensity, Duration)

Example: An intense pleasure is more valuable than a less intense one whereas a pain of a given intensity is worse the longer it lasts.

Estimating Value of Certain Interest:
Total Suffering = Average Suffering per unit time * amount of Time
Average Suffering per unit time = (Suffering per unit time at the beginning of the period + Suffering per unit time at the end of the period) / 2

Extent

Extent of the Interest = Number of individuals to which interest applies.
Consider all interests equally. "Each counts for one, and none for more than one."

Model of Above is as follows:
Total Suffering = Average Suffering per individual * Number of Individuals

If interest is different for each person, then:
Total Suffering = Suffering of 1st individual + Suffering of 2nd individual + Suffering of 3rd + ...

Certainty

This factor calculates the value of a given action before we have performed it. It uses Set Theory, Graph Theory and mainly Probability Theory.

An example:

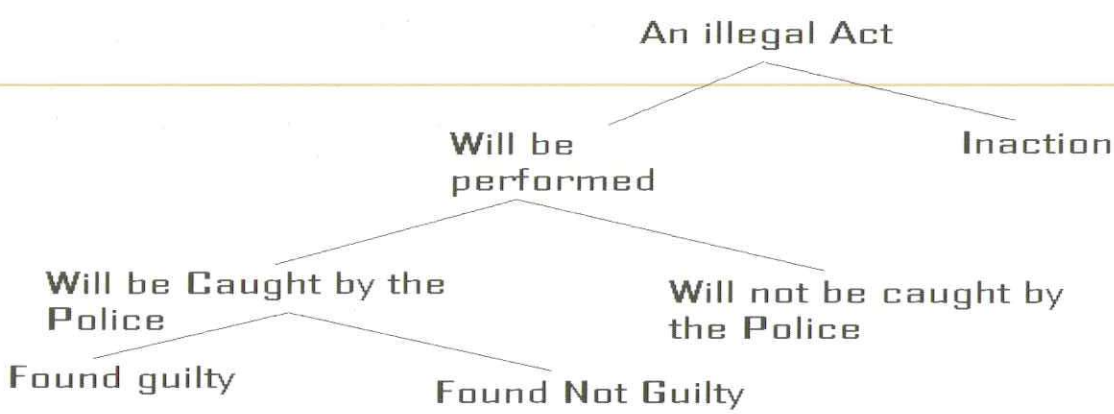

Fig.1: Tree Diagram Demonstrating an Action

Dictum of Great Philosophers and Mathematicians

French Philosopher & Mathematician, Rene Descartes said:
"With me, everything turns into Mathematics!"

Pythagoras tried to bring Mathematical order into Ethical field. He asserted that: "Justice is represented by a Square number."

Plato and Aristotle showed close relationship of the good and the beautiful. According to them, M(Ethical Measure) = G (Total Good Achieved). They regarded: "Beauty is characterized by UNITY in variety."

One of the greatest Islamic scholars, Hadhrat Ali, defined kinds of people [3] as follows:

- Learned people who are highly versed in the ethics of truth.
- Those who are acquiring the above knowledge.
- Class of people who are uneducated. They follow every pretender and accept every slogan, they have neither acquired any knowledge nor have they secured any support of firm and rational convictions.

According to him, if sensible trustees of knowledge and wisdom totally disappear from human society then both knowledge and wisdom will suffer severely, may bring harm to humanity and may even die out.

Can't Mathematics SERVE here?

After viewing what kinds of people are and how Mathematics is used for performing actions, we propose that Math can be used for character building. We produce some examples here that may serve math educators for making their lectures fruitful for future:

First – Degree Equations in one-variable

In an introductory class of "Business Mathematics", types of first-degree equations in one-variable as identities, conditional equations and contradictions (false statements) are discussed. What we define usually here, an identity is an equation that is true for all values of a variable, and therefore an identity has infinitely many solutions. A conditional statement is only true for a limited number of values of the variable and thus it has unique solution where as a contradiction is never true and therefore it has no solution [4].

Teaching as it is in the textbook is sometimes really hard. A teacher has to make the concepts relevant to the student in his real life so as to define a role for a student that he can play in society and this is the best place where a learner of mathematics can understand that mathematics teaches honesty. As for example, a contradiction says that $x + 5 = x + 3 \Rightarrow 5 = 3$ is impossible because 5 is 5 and 3 is 3. Mathematics gives distinction between right and wrong. It can make 3, as 5 by addition of 2 or 5 as 3 by subtracting 2 from 5 but will never call 5 equals to 3. This is what in general, we humans should learn. We cannot make horses equal to donkeys till the time we do not subtract some properties of horses.

Limits

Cutting off the details [3] limits can be visualized as behavior of people. When Mr. X has an extremely bad behavior due to anger, Mr. Y just smiles and gives a cool answer that is the place where limit "X" tends to infinity over "Y" exists and we say horizontal asymptote exists. Similar is the case of vertical asymptotes but when Mr. X is in anger, Mr. Y also becomes angry, limits do not exist as our mind is out of control in anger. This is the way I enjoyed teaching limits in my classroom and it made students learn limits quickly as compared to the traditional way.

Distributive law in "Near rings"

Near-rings are generalized rings, defined as algebraic structures with (R, +) not necessarily an abelian group, (R, .) a semi group with just one distributive property, either right or left. Distributive near-rings are those near rings, which still have (R, +) just a group but with both distribute properties. If R is a distributive near ring then $(R^2, +)$ is abelian. If $1 \in R$ and R is a distributive near ring, then R is a ring. For details, one can read (7) and (8). Whereas rings are those algebraic structures which have (R, +) an abelian group, (R, .) a semi group with both left and right distributive laws.

Once one of my students sent me a math joke in a message on my mobile phone. The text of the joke was:

"Study = Do not fail -------- Equation [1]
Do not study = fail --------- Equation [2]

Adding equations [1] and [2], we get
Study + do not study = do not fail + fail
⇨ Do not study + study = Do not fail + fail
⇨ (Do not + 1) study = (Do not +1) fail
⇨ Study = fail
So why should we study."

As a learner of mathematics, I observed, the result of the joke was true for algebraic structure of rings but interestingly, false for the near-rings, even if we either consider R to be a distributive near ring (obviously $1 \notin R$) or consider R to be a near ring containing 1. Here I had to consider that study, do not study, fail, do not fail are elements of a near ring.

This is what we learn in our life too that if we continue traveling on the right path and we stick to the right path, we reach our destination some day but if we make on jumping from one way to another non-seriously, from right to left, left to right, we can really face results as in the joke!

Concluding Remarks & Future Work

Mathematics is a rich and fruitful discipline. Researchers having interest in Mathematics Education can further work on usage of "Mathematics" in character building and ethics so as to bring betterment in the society using both knowledge and wisdom.

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