MATHEMATICAL MODEL FOR SEEEKING OF AN OPTIMAL PATH

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Abstract

Nazimabad Chest Hospital (Ojha) is the well known Hospital for the best treatment of T.B patients among other three Hospitals in Karachi. This hospital is Located within the premises of Abbasi Shaheed Hospital Nazimabad No. 3 Karachi, and has modern equipment and qualified staff for treatment of TB patients. The patients have to visit the Hospital periodically in between one to four or more weeks, from their homes/ residence. They uses different routes to reach the destination, since the hospital has different approaches therefore the patients living Malir Cantt, North Karachi, Gulshan-e-iqbal etc areas need a path/ route which is to be shortest and take less time and subsequently reduce the transportation charges. In order to facilitate and to address their difficulties an effort is being made to find out the shortest path. A mathematical model with the help of Floyd's algorithm is being developed to achieve the desired objective.

Keywords: *Floyd algorithm, Node 1-Malir Cantt, Node 5- Nazimabad Chest Hospital Karachi (OHJA) (at Abbasi Shaheedt hospitalad) Permanent label, Backtracking*

Introduction:

Nazimabad OHJA Insitute of Chest Hospital is located in the premises of Abbasi Shaheed Hospital at Nazimabad. The said hospital is the only hospital having modern facilities and competent staff comprising doctors and paramedical staff available for the people residing with in Karachi city and near by areas such as Gulsahn-e-Iqbal, Malir Cantt ,North Karachi, North Nazimabad etc. Patient living in far-flung areas are facing difficulties in getting medical treatment from institute of chest hospital, for the purpose, they spend a lot of time, money and also has to travel, large distances to reach Hospital.

In order to provide ease and to facilitate the needy and poor patient an effort has been made to find out an optimal path, (i.e- the shortest route from Malir cantt to Naizmabad OHJA chest hospital. To achieve the objective the city government, Highway Authority and concerned department have been approached to get various information required for determining the optional path, are appended below.

- Distance from Malir(Node 1) To NIPAChowrangi(Node 2)d12 =10 Km
- Distance from NIPA (Node 2)To Laiquatabad Node 4 =7 Km
- Distance from Malir Cantt (Node 1) To Gulshan Chowrangi (Node 3) d13=12 Km
- Distance from Gulshan Chowrangi(Node 3) To Laiquatabad (Node4) d34=10km
- Distance from Gulshan Chowrangi (Node 3)To Abbasi Shaheed Node (5) d35 =13km

To find the shortest route from Malir Cantt to Insituate Of Chest Hospital Nazimabad the Floyd's algorithm is considered approperiate for the purpose. Floyd's algorithm is designed to calculate the shortest route / path between the source node and every other node in the network analysis. Floyds Algorithm is more general because it allows the determination of the shortest route / path between any two nodes in the network

The algorithm represent n- node network as a square matrix having n rows and n column, , entry (i,j)

Methodology And Modelling:

In the present case the Floyd's Algorithm is considered to be suitable for finding the shortest path, the Floyd's algorithm is stated as follows

The algorithm represent an n-node network as square matrix with n-rows and a n-Columns entry (i,j) of the matrix gives the distance dij from the node i to j which is finite if i is linked directly to j and infinite otherwise. Given three nodes i, j and k as shown in Fig A with the connecting distances shown by the three arcs, it is shorter to reach k from i passing through j if

$d_{ij}+d_{jk} < d_{ik}$

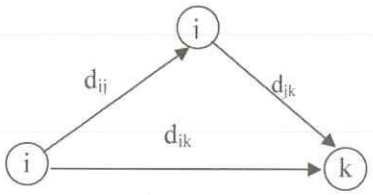


Fig A

It is optimal to replace the direct route from i → k with the indirect route i → j → k. This tripple operation is applied symmetrically to the network using the following steps

Step 0: Defining the starting distance matrix D_0 and node sequence matrix $S_{0,.}$. The diagonal elements are marked with (--) to indicate that they are blocked. Set k=1

Step k: Define the row k and column k as pivot row and pivot column. Apply the tripple operation each d_{ij} in D_{k-1} for all i and j. if the condition.

$d_{ik}+ d_{kj} < d_{ij}$,$(i \neq k , j \neq k)$

is satisfied, make the following changes .

(a) Create D_k by replacing d_{ij} in D_{k-1} with $d_{ik}+ d_{kj}$.
(b) Create S_k by replacing S_{ij} in S_{k-1} with k set k=k+1 and repeated step k.

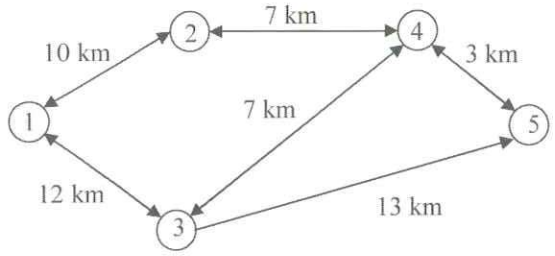
After 1 2 3 … n, n step we can determine the shortest route/ path between node i and j from the matrices

D_n and S_n using the following,

(1) From D_n , d_{ij} give the shortest distance between the node i and j .

(2) From S_n, determine the intermediate node k $=S_{ij}$, which yields the route $i \rightarrow k \rightarrow j$,

If S_{ik} =k and S_{kj} = j stop all the intermediate node of the route have been found other wise repeat the procedure between the node i and k, and node k and j . For the present scenario the network diagram is as fallows.



Node 1 Malir Cantt
Node 2 Nipa
Node 3 Gulshan Chowranging
Node 4 Laiquatabad
Node 5 Nazimabad Chest Hospital

Iteration k=0

The matrices D0 and S0 give the intial representation of the network, D0 is symmetric except that $d_{53} = 8$ because no traffic is allowed from node 5 to node 3.

D_0

	1	2	3	4	5
1	--	10	12	∞	∞
2	10	--	∞	7	∞
3	12	∞	--	10	13
4	∞	7	10	--	3
5	∞	∞	∞	3	--

S_0

	1	2	3	4	5
1	--	2	3	4	5
2	1	--	3	4	5
3	1	2	--	4	5
4	1	2	3	--	5
5	1	2	3	4	--

Iteration k=1

The pivot row and pivot column are given by the first row and first column (k=1), as Shown in the D_0 Matrix the element d23 and d32 are the only one that can be improved by the tripple operation thus to obtain D_1 and S_1 from D_0 and S_o

Replace d_{23} with $d_{23} = d_{21} + d_{13} = 10 + 12 = 22$ and set $S_{23} = 1$

Replace d_{32} with $d_{32} = d_{31} + d_{12} = 12 + 10 = 22$ and the set $S_{32} = 1$

D_1

	1	2	3	4	5
1	--	10	12	∞	∞
2	10	--	22	7	∞
3	12	22	--	10	13
4	∞	7	10	--	3
5	∞	∞	∞	3	--

S_1

	1	2	3	4	5
1	--	2	3	4	5
2	1	--	1	4	5
3	1	1	--	4	5
4	1	2	3	--	5
5	1	2	3	4	--

Iteration k=2

The tripple operation is applied in D_1 and S1 the change are shown in D_2 and S_2

Replace $d_{14} = d_{12} + d_{24} = 10 + 7 = 17$ $S_{14} = 2$

Replace $d_{41} = d_{42} + d_{21} = 7 + 10 = 17$ $S_{41} = 2$

D_2

	1	2	3	4	5
1	--	10	12	17	∞
2	10	--	22	7	∞
3	12	22	--	10	13
4	17	7	10	--	3
5	∞	∞	∞	3	--

S_2

	1	2	3	4	5
1	--	2	3	2	5
2	1	--	1	4	5
3	1	1	--	4	5
4	2	2	3	--	5
5	1	2	3	4	--

Iteration k=3

The tripple operation is applied in D2and S2 the change are shown in D_3 and S_3

Replace d25 with $d_{25} = d_{24} + d_{45} = 7 + 3 = 10$ and the set $S_{52} = 3$

Replace d52 with $d_{52} = d_{54} + d_{42} = 3 + 7 = 10$ and set $S_{25} = 3$

Replace d_{15} with $d_{15} = d_{35} + d_{12} = 12 + 13 = 25$ and the set $S_{15} = 3$

D_3

	1	2	3	4	5
1	--	10	12	17	25
2	10	--	22	7	10
3	12	22	--	10	13
4	17	7	10	--	3
5	∞	10	∞	3	--

S_3

	1	2	3	4	5
1	--	2	3	2	3
2	1	--	1	4	3
3	1	1	--	4	5
4	2	2	3	--	5
5	1	3	3	4	--

Iteration k=4

Set k=4 as shown by row and column in D3 the new matrices re given by D_4 and S_4

Replace d_{15} with $d_{15} = d_{14}$ $(=d_{12}+d_{24}) + d_{45} =$ $17(=10+7) + 3 = 20$ and set $S_{15} = 4$

Replace d_{51} with $d_{51} = d_{54} + d_{41}(=d_{42}+ d_{21}) =$ $3 + 17 = 20$ and the set $d_{51} = 4$

Replace d_{53} with $d_{53} = d_{54} + d_{43} = 3 + 7 = 10$ and set $d_{53} = 4$

Replace d_{35} with $d_{35} = d_{34} + d_{45} = 7 + 3 = 10$ and set $d_{35} = 4$

Replace d_{23} with $d_{23} = d_{24}+ d_{43} = 7 + 7 = 14$ and set $S_{23} = 4$

Replace d_{32} with $d_{32} = d_{34}+ d_{42} = 7+7 = 14$ and the set $S_{32} = 4$

D_4

	1	2	3	4	5
1	--	10	12	17	20
2	10	--	14	7	10
3	12	14	--	10	10
4	17	7	10	--	3
5	20	10	10	3	--

S_4

	1	2	3	4	5
1	--	2	3	2	4
2	1	--	4	4	4
3	1	4	--	4	4
4	2	2	3	--	5
5	4	4	4	4	--

Iteration k=5

Set k=5 Since no improvement are possible in matrices D4 and S4 therefore the process may be stopped.

RESULTS AND DISCUSSION

The shortest route that is optimal path from node 1 (Malir Cantt).to node 5 (Institute Nazimabad Chest Hospital In Abbasi Shaheed Hospital) has been calculated with the help of Floyd's algorithm is equal to20 km, the same is evident from iteration 4 which is appended below:

D_4

	1	2	3	4	5
1	--	10	12	17	20
2	10	--	14	7	10
3	12	14	--	10	10
4	17	7	10	--	3
5	20	10	10	3	--

S_4

	1	2	3	4	5
1	--	2	3	2	4
2	1	--	4	4	4
3	1	4	--	4	4
4	2	2	3	--	5
5	4	4	4	4	--

The final matrices D4 and S4 contain all the information needed to determine the shortest route between any two nodes in the network. Therefore the shortest path from MalirCantt (Node 1) to chest Hospital Nazimabad (Node5) is $d_{15} = 20$ km

To determine associated route consider the segment (i,j) that represents a direct link If

$S_{ij} = j$

But in S_4 $S_{15} = 4 \neq 5$ This shows that there is an intermediate node between 1---- 5 that is 4 that links node1 to node 5 that is

$$1 \longrightarrow 4 \longrightarrow 5$$

Now Similarly $S_{14} = 2 \neq 4$ therefore

The segment 1-4 is not directly linked.

Hence there is another intermediate node between 1-4 i.e. 2

The route $1 \longrightarrow 4$ is replaced by $1 \longrightarrow 2 \longrightarrow 4$

Because $S_{12} = 2$, $S_{24} = 4$, no further intermediate nodes exist

So the route from Node 1(Malir Cantt) to Node 5 (Nazimabad Chest Hospital) is $1 \longrightarrow 2 \longrightarrow 4 \longrightarrow 5$ and the shortest path obtained is d_{15} = 20 Km

CONCLUSION:

In order to facilitate the T.B. patients visiting institute of chest hospital Nazimabad No.3 from different areas in Karachi a shortest path/route between MalirCantt and Naizmabad institute of chest hospital has been calculated , which is $d_{15} = 20$ km with the help of Floyd's algorithm. however the route include, the node point 1,2,4,5 (i.e) patients coming from Malir Cantt are to follow the route as under

MalirCantt → NIPA → Laiquatabad → Nazimabad Chest Hospital. The route is said to be the shortest path and distance covered by the individual will following the said route be 20Km

References:

1 Hillier Lieberman, Introduction to operation Research. Edition 8, 2005

2 Premkumar Gupta D.S. Hera, Operation Research

3 J.P.Brams, operational reseach '84 Edition 1984

4 Joseph G.ecker Michael kupferschanid 1988

5 Hamdy- A- Taha Operation Research Edition 8, 2008.