H_∞ MODEL REFERENCE ADAPTIVE CONTROL FOR ROBOT MANIPULATORS

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ABSTRACT

ABSTRACT: A good trajectory tracking of robotic system is very important. In this paper, a model reference adaptive control (MRAC) containing adaptive error gain with H_∞ performance is proposed to achieve robust trajectory tracking of n-link robotic manipulator. In the presented method, the adaptive control is used to stabilize the system, compensate the parameter uncertainty, and vanish the tracking errors significantly. The stability analysis is accomplished for the proposed control scheme using Lyapunov function and linear matrix inequality (LMI). Simulation results show that the effectiveness of the proposed method. It also shows that proposed scheme can obtain robust trajectory tracking with a desired H_∞ performance γ.

Key Words: Adaptive control, H_∞ performance analysis, Asymptotic stability

1. INTRODUCTION

Uncertain dynamics of robotic manipulator is a highly nonlinear and parameter variant and it is difficult to stabilize and control. This system is mostly used in medical, military and industrial applications. Manipulator can also be considered for the biped model and exoskeleton device, whereas exoskeleton is a wearing robotic device used for rehabilitation of patients, and military purposes.

Adaptive control is a renowned scheme and it is extensively developed to control robotic systems, aerial vehicles, autonomous vessels and many other industrial applications [1-10]. MRAC is an adaptive controller and its parameters are adjusted by adaptation law; thus the adaptive gain may lead to satisfactory control performances for the system.

Numerous robotic plants have been controlled by the adaptive controller. For 2-dof robotic manipulator, prominent adaptive control schemes were proposed in [1-3]. In [9], an adaptive control method was presented with feedback and feedforward controllers for the robotic manipulator to control the state and torque, whereas the Lyapunov candidate ensures the asymptotic stability of closed-loop system. To achieve the good performance of trajectory tracking, some other robotic applications (biped model, orthosis and exoskeleton) were also controlled via adaptive technique [11-13].

To obtain high trajectory tracking performance of the system, significant efforts have been done seeking the new control approaches. The robust scheme of H∞ control was proposed by [14] and has been widely used to control the system dynamics to mitigate the unknown disturbances. H∞ control technique is widely used to reduce the influence of perturbations on the system [15]. In [16], H∞ control was used for uncertain nonlinear systems and it was also employed to control the applications such as the robust control, optimal control, and linear and nonlinear observers. Moreover, H∞ with Kalman and unscented Kalman filters have been developed to obtain robustness against uncertainties and state estimation, simultaneously [5, 8, 10]. In addition, H∞ control was effectively used to systems, for example robotic applications, 2-D and seismic-excited systems etc. [15,17,18,19].

H∞ is labeled as high robustness and its behavior is insensitive to disturbances. Different type of control schemes were proposed [20,21] with H∞ control, to deal with internal asymptotic stability of the system. One of the problems was linked with adaptive control, that the trajectory tracking was not achieved in the presence of disturbance. Researchers preferred H∞ technique to attenuate the external disturbance. Although some authors combined H∞ technique with adaptive control to get robust trajectory tracking of the manipulator [22,23].

Tracking and robustness is the most important task in robot applications. Robotic systems are controlled by a variety of robust tracking control methods [24]. In previous work [25], trajectory tracking and stability of exoskeleton were achieved by MRAC with adaptive error gain in the presence of disturbance. In this paper, adaptive control technique in [23] and MRAC method are combined together with H∞ performance then used for manipulator to achieve robustness, parameter update and trajectory tracking. The controller gains and parameter updating are obtained by linear matrix inequality (LMI) as well as adaptation law. Here State gain, reference gain and unknown parameter of system are auto-tuned by proposed adaptive control, which provides good tracking. In which robustness is obtained by H∞ performance, which is attenuating the effect of unknown external disturbance. Moreover, a suitable Lyapunov candidate is developed, which ensures the system's asymptotic stability and H∞ performance for robotic manipulator under parametric uncertainties and external disturbances.

This paper is ordered as follows: Section 2 explains the system dynamics. In section 3, the proposed adaptive scheme with H∞ performance and asymptotic stability analysis are illustrated. Section 4 provides an example and simulation results. Finally, the conclusion is contained in section 5.

2. System Dynamics

Before beginning the detailed discussion, the given notations will be used throughout the article. M is a symmetric and positive definite matrix, $\eta_{\min}(M(\theta)) \le \|M(\theta)\| \le \eta_{\max}(M(\theta))$, where $\eta_{\min}$ and $\eta_{\min}$ denote eigenvalues of $M(\theta)$. The transpose is denoted as superscript T, identity matrix is expressed as I and the symmetric term in LMI is denoted by asterisk '*'. The L_2-norm of the unknown disturbance is as follows

$$\|w\|_2 = \sqrt{\int_0^\infty w^T(t)w(t)dt}$$

where $w(t) \in L_2[0,\infty)$, $if\ \|w\|_2 < \infty$.

From [1], dynamic equation of the n-link robotic manipulator is presented as

$$M(\theta)\ddot{\theta} + H(\theta,\dot{\theta})\dot{\theta} + G(\theta) = \tau + w \qquad (1)$$

where θ is the vector of angular position and used to determine the movement of joints, $M(\theta)$ is the inertia matrix, $H(\theta,\dot{\theta})$ is centripetal and coriolis forces vector, is the gravitational force vector, $G(\theta)$ represent the control input, and w denotes the unknown lumped disturbance.

From Eq. (1), one can write as

$$\ddot{\theta} = M^{-1}(\theta)(\tau + w) - M^{-1}(\theta)[H(\theta,\dot{\theta})\dot{\theta} + G(\theta)] \quad (2)$$

Let,

$$M^{-1}(\theta)(H(\theta,\dot{\theta})\dot{\theta} + G(\theta)) = Y(\theta,\dot{\theta})\phi$$

Then

$$\Rightarrow \ddot{\theta} = M^{-1}(\theta)(\tau + w) - Y(\theta,\dot{\theta})\phi \quad (3)$$

In the above equation, $Y(\theta,\dot{\theta})$ presents the regression matrix and ϕ denotes the vector of unknown parameters. Moreover, $Y(\theta,\dot{\theta})$ contains the known system structure with uncertain parameters.

3. Main Results

3.1 Adaptive Control Design

In this subsection, the designing of the proposed adaptive controller and the stability analysis using appropriate Lyapunov functional candidate will be presented.

For system Eq. (3), control torque is chosen as,

$$\tau = \hat{M}(\theta)[\hat{p}^T x + \hat{q}^T r + u + Y(\theta,\dot{\theta})\hat{\phi}] \quad (4)$$

where, $u = Ke$ is the control input, the estimation of the inertia matrix M is denoted by $\hat{M}$, estimation of ϕ is represented by $\hat{\phi}$, state and reference adaptation parameters are represented by $\hat{p}$ *and* $\hat{q}$, respectively.

Substitution of Eq. (4) into Eq. (3), one obtains

$$\ddot{\theta} = M^{-1}(\theta)\left\{\hat{M}(\theta)\left[\hat{p}^T x + \hat{q}^T r + u + Y(\theta,\dot{\theta})\hat{\phi}\right]\right. \quad (5)$$

$$\left. + M^{-1}(\theta)w - Y(\theta,\dot{\theta})\phi\right.$$

If perfect modeling is supposed, then the parameters of actual model and estimated model should be identical [26], therefore $\hat{M}(\theta) = M(\theta)$.

$$\Rightarrow \ddot{\theta} = \hat{p}^T x + \hat{q}^T r + u + M^{-1}(\theta)w + Y(\theta,\dot{\theta})\tilde{\phi} \quad (6)$$

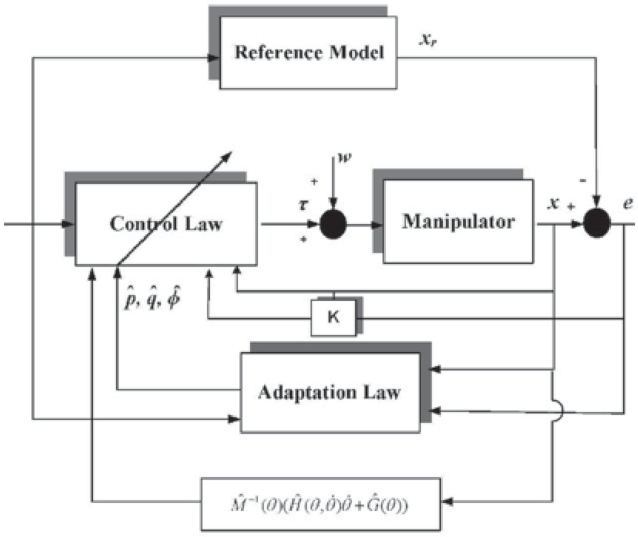

Fig. 1: Proposed model structure

Now Eq. (6), it can be expressed in the form of state equation as

$$\dot{x} = Ax + B[\hat{p}^T x + \hat{q}^T r + u + M^{-1}w + Y\tilde{\phi}] \quad (7)$$

Where

$$A = \begin{bmatrix} 0 & I_{m\times m} \\ 0 & 0 \end{bmatrix}_{n\times n}, \quad B = \begin{bmatrix} 0 \\ I_{m\times m} \end{bmatrix}_{n\times m} \quad and \quad x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$$

The reference model with the desired trajectory can be written as follows

$$\dot{x}_r = A_r x_r + B_r r \quad (8)$$

where $A_r \in \mathbb{R}^n$ and $B_r \in \mathbb{R}^m$ are known matrices, A_r is Hurwitz matrix and $r(t) \in \mathbb{R}^m$ denotes the bounded reference. Suppose that the ideal gains exist, which are satisfying matching model condition $A + Bp^T = A_r$ and $Bq^T = B_r$.

Now, the tracking error vector and its derivatives can be written as follows

$$e = x - x_r \quad \Rightarrow \dot{e} = \dot{x} - \dot{x}_r \quad (9)$$

where x and x_r is the actual and desired state vectors, respectively.

By substituting Eq. (7) and Eq. (8) into Eq. (9), one can obtain the tracking error as follows

$$\dot{e} = \left[A + B\hat{p}^T \right] x + B\hat{q}^T r + Bu + BM^{-1} w + BY\widehat{\phi}$$
$$-A_r x_r - B_r r \pm A_r x$$

$$\Rightarrow \dot{e} = A_r e + \left[A + B\hat{p}^T - A_r \right] x + B\hat{q}^T r - Bq^T r + Bu$$
$$+BM^{-1} w + BY\widehat{\phi}$$

$$e \quad A_r e \quad B(\hat{p}^T \quad p^T)x \quad B(\hat{q}^T \quad q^T)r \quad Bu$$

$$BM \;{}^1 w \quad BY \tag{10}$$

where $p^T \quad \hat{p}^T \quad p^T$ and $q^T \quad \hat{q}^T \quad q^T$ are the difference between adaptive gains and constant gains. Then Eq. (10) will be,

$$\dot{e} = A_r e + B(\widehat{p}^T x + \widehat{q}^T r + u + M^{-1} w + Y\widehat{\phi}) \tag{11}$$

$$z = Ce + Du + Ew \tag{12}$$

where Eq. (11) is closed loop error equation. The controlled output is denoted by $z \in \mathbb{R}^p$. The constant matrices C, D and E have the suitable dimensions.

Lemma 1*. Assume a following square matrix*

$$G = \begin{bmatrix} L & F \\ F^T & R \end{bmatrix}, \textit{ with the given conditions } [27]$$

i. $G < 0;$
ii. $L < 0, R - F^T L^{-1} F;$
iii. $R < 0, L - FR^{-1} F^T.$

Definition 1. Closed-loop system (11) has a decided γ-disturbance attenuation performance. When disturbance is equal to zero, it is robustly asymptotically stable. Moreover, the following condition under zero boundary condition given as

$$\|z\|_2^2 < \gamma^2 \|w\|_2^2, w(t) \in L_2[0, \infty)$$

To obtain significant results, the Lyapunov function can be considered for the stability analysis and adaptation law can be formulated for the robotic manipulator simultaneously. Therefore, the following Theorem gives an appropriate condition that the system (11) is robustly asymptotically stable in the absence of disturbance.

Theorem 1: *With* w=0, *system (11) is robustly asymptotically stable if positive definite symmetric matrices P,* Λ_p, Λ_q, and Λ_ϕ *exist with suitable dimensions under adaptation laws as follows*

$$\dot{\hat{p}} = -\Lambda_p x e^T PB$$
$$\dot{\hat{q}} = -\Lambda_q r e^T PB \tag{13}$$
$$\dot{\hat{\phi}}^T = -\Lambda_\phi e^T PBY$$

Proof: The following Lyapunov functional is selected

$$V(e, \widehat{p}, \widehat{q}, \widehat{\phi}) = e^T Pe + tr\left(\widehat{p}^T \Lambda_p^{-1} \widehat{p}\right) + tr\left(\widehat{q}^T \Lambda_q^{-1} \widehat{q}\right)$$
$$+\widehat{\phi}^T \Lambda_\phi^{-1} \widehat{\phi} \tag{14}$$

with $\Lambda_p^T = \Lambda_p > 0, \Lambda_q^T = \Lambda_q > 0 \textit{ and } \Lambda_\phi^T = \Lambda_\phi > 0.$

Derivative of Eq. (14) can be written as,

$$\dot{V}(e, \widehat{p}, \widehat{q}, \widehat{\phi}) = e^T P\dot{e} + \dot{e}^T Pe + tr\left(\widehat{p}^T \Lambda_p^{-1} \dot{\widehat{p}} + \dot{\widehat{p}}^T \Lambda_p^{-1} \widehat{p}\right)$$
$$+tr\left(\widehat{q}^T \Lambda_q^{-1} \dot{\widehat{q}} + \dot{\widehat{q}}^T \Lambda_q^{-1} \widehat{q}\right) + \widehat{\phi}^T \Lambda_\phi^{-1} \dot{\widehat{\phi}} + \dot{\widehat{\phi}}^T \Lambda_\phi^{-1} \widehat{\phi}$$

$$\Rightarrow \dot{V}(e, \widehat{p}, \widehat{q}, \widehat{\phi}) = e^T P\left(A_r e + B\widehat{p}^T x + B\widehat{q}^T r + Bu + BY\widehat{\phi}\right)$$
$$+\left(A_r e + B\widehat{p}^T x + B\widehat{q}^T r + Bu + BY\widehat{\phi}\right)^T Pe$$
$$+tr\left(2\widehat{p}^T \Lambda_p^{-1} \dot{\widehat{p}}\right) + tr\left(2\widehat{q}^T \Lambda_q^{-1} \dot{\widehat{q}}\right) + 2\dot{\widehat{\phi}}^T \Lambda_\phi^{-1} \widehat{\phi}$$

$$\Rightarrow \dot{V}(e, \widehat{p}, \widehat{q}, \widehat{\phi}) = e^T \left(PA_r + A_r^T P\right) e + 2e^T PB\widehat{p}^T x$$
$$+tr\left(2\widehat{p}^T \Lambda_p^{-1} \dot{\widehat{p}}\right) + 2e^T PB\widehat{q}^T r + tr\left(2\widehat{q}^T \Lambda_q^{-1} \dot{\widehat{q}}\right)$$
$$+e^T PBKe + e^T K^T B^T Pe + 2e^T PBY\widehat{\phi} + 2\dot{\widehat{\phi}}^T \Lambda_\phi^{-1} \widehat{\phi}$$

$$\Rightarrow \dot{V}(e, \widehat{p}, \widehat{q}, \widehat{\phi}) =$$
$$e^T \left(PA_r + A_r^T P + PBK + K^T B^T P\right) e$$
$$+2tr\left[\widehat{p}^T (xe^T PB + \Lambda_p^{-1} \dot{\widehat{p}})\right] \tag{15}$$
$$+2tr\left[\widehat{q}^T (re^T PB + \Lambda_q^{-1} \dot{\widehat{q}})\right]$$
$$+2\left[e^T PBY + \dot{\widehat{\phi}}^T \Lambda_\phi^{-1}\right] \widehat{\phi}$$

Therefore, by substituting (13) into (15), one has

$$\dot{V}(e, \widehat{p}, \widehat{q}, \widehat{\phi}) = e^T \left(P(A_r + BK) + (PA_r + BK)^T P\right) e$$

(16)

where $P(A_r + BK) + (PA_r + BK)^T P < 0$.

One can rewrite above eq. in the form LMI as

$$\Rightarrow \; PA_r + PBK + A_r^T P + K^T B^T P < 0$$

pre and post multiply by $\{P^{-1}\}$

$$\Rightarrow \; A_r P^{-1} + BKP^{-1} + P^{-1} A_r^T + P^{-1} K^T B^T < 0$$

Let $P^{-1} = \bar{P}$ and $L = K\bar{P}$

$$\Rightarrow \; A_r \bar{P} + BL + \bar{P} A_r^T + L^T B^T < 0$$

It is implies that $\dot{V}(e, \hat{p}, \hat{q}, \hat{\phi}) < 0$.

Based on the above equation, the closed-loop system is asymptotically stable. This completes the proof.

Remark 1:

In order to improve tracking performance with parameter uncertainties, proposed adaptive law is introduced. In the control design, a Lyapunov functional candidate is selected as in [4] with additional terms of error gain K and parameter variation vector $\hat{\phi}$.

3.2 H∞ Performance Analysis

This section presents the H∞ disturbance attenuation performance of Eq. (11) with an appropriate condition in Theorem 2.

Theorem 2:

System Eq. (11) is robustly asymptotically stable with positive (constant) value of γ, if there exists symmetric positive definite matrix P and error gain K with suitable dimension, such that the given LMI holds

$$\begin{bmatrix} A_r\bar{P} + \bar{P}A_r^T + BL + L^T B^T & BM^{-1} & L^T D^T + \bar{P} C^T \\ * & -\gamma^2 & E^T \\ * & * & -I \end{bmatrix} < 0 \qquad (17)$$

where, $L = K\bar{P}$.

Proof: Now we will prove the system Eq. (11) with any non-zero $w \in L_2[0,\infty)$ has a decided γ-disturbance attenuation performance. Let us introduce

$$\Pi = \dot{V}(e, \hat{p}, \hat{q}, \hat{\phi}) + z^T(t) z(t) - \gamma^2 w^T(t) w(t) < 0 \qquad (18)$$

By substituting Eq. (12) in Eq. (18), one has

$$\Pi = e^T P \dot{e} + \dot{e}^T P e + tr\left(\hat{p}^T \Lambda_p^{-1} \dot{\hat{p}} + \dot{\hat{p}}^T \Lambda_p^{-1} \hat{p}\right)$$
$$+ tr\left(\hat{q}^T \Lambda_q^{-1} \dot{\hat{q}} + \dot{\hat{q}}^T \Lambda_q^{-1} \hat{q}\right) + \hat{\phi}^T \Lambda_\phi^{-1} \dot{\hat{\phi}} + \dot{\hat{\phi}}^T \Lambda_\phi^{-1} \hat{\phi}$$
$$+ (Ce + Du + Ew)^T (Ce + Du + Ew) - \gamma^2 w^T w < 0$$

$$\Rightarrow \Pi = e^T P\left(A_r e + B(\hat{p}^T x + \hat{q}^T r + Y\hat{\phi} + M^{-1} w + u)\right)$$
$$+ \left(A_r e + B(\hat{p}^T x + \hat{q}^T r + Y\hat{\phi} + M^{-1} w + u)\right)^T Pe$$
$$+ tr\left(2\hat{p}^T \Lambda_p^{-1} \dot{\hat{p}}\right) + tr\left(2\hat{q}^T \Lambda_q^{-1} \dot{\hat{q}}\right) + 2\dot{\hat{\phi}}^T \Lambda_\phi^{-1} \hat{\phi}$$
$$+ e^T (C + DK)^T (C + DK) e + e^T (C + DK)^T Ew$$
$$+ w^T E^T (C + DK) e + w^T E^T Ew - \gamma^2 w^T w < 0$$

$$\Rightarrow \Pi = e^T \left(PA_r + A_r^T P + (C + DK)^T (C + DK) + PBK + K^T B^T P\right) e$$
$$+ 2tr\left[\hat{p}^T (xe^T PB + \Lambda_p^{-1} \dot{\hat{p}})\right] + 2tr\left[\hat{q}^T (re^T PB + \Lambda_q^{-1} \dot{\hat{q}})\right]$$
$$+ 2\left[e^T PBY + \dot{\hat{\phi}}^T \Lambda_\phi^{-1}\right]\hat{\phi} + e^T \left(PBM^{-1} + (C + DK)^T E\right) w$$
$$+ w^T \left(M^{-1^T} B^T P + E^T (C + DK)\right) e + w^T \left(E^T E - \gamma^2\right) w < 0$$

Substitution of Eq. (13) into above equation, one obtains

$$\begin{aligned} \Pi = {} & e^T\left(PA_r + A_r^T P + (C+DK)^T (C+DK) + PBK + K^T B^T P\right) e \\ & + e^T\left(PBM^{-1} + (C+DK)^T E\right) w \\ & + w^T\left(M^{-1^T} B^T P + E^T (C+DK)\right) e \\ & + w^T\left(E^T E - \gamma^2\right) w < 0 \end{aligned} \qquad (19)$$

Now one can write Eq. (19) in the form of LMI,

$$\Pi = \begin{bmatrix} e \\ w \end{bmatrix}^T \Omega \begin{bmatrix} e \\ w \end{bmatrix}$$

where,

$$\Omega = \begin{bmatrix} PA_r + A_r^T P + (C+DK)^T (C+DK) + PBK + K^T B^T P & PBM^{-1} + (C+DK)^T E \\ * & E^T E - \gamma^2 \end{bmatrix} < 0$$

By applying Lemma 1, one gets

$$\Rightarrow\begin{bmatrix} PA_r + A_r^T P + PBK + K^T B^T P & PBM^{-1} & (C+DK)^T \\ * & -\gamma^2 & E^T \\ * & * & -I \end{bmatrix} < 0$$

pre and post multiply by $diag\left\{P^{-1}, I, I\right\} \Rightarrow \bar{P} = P^{-1}$

$$\Rightarrow\begin{bmatrix} A_r\bar{P}+\bar{P}A_r^T+BK\bar{P}+\bar{P}K^TB^T & BM^{-1} & \bar{P}K^TD^T+\bar{P}C^T \\ * & -\gamma^2 & E^T \\ * & * & -I \end{bmatrix}<0$$

(20)

By integrating Eq. (18), it can be obtained that

$$\int_0^\infty \dot{V}(e,\tilde{p},\tilde{q},\tilde{\phi}) + \int_0^\infty z^T z\, dt - \gamma^2 \int_0^\infty w^T w\, dt < 0$$

$$\Rightarrow \int_0^\infty z^T z\, dt - \gamma^2 \int_0^\infty w^T w\, dt < \int_0^\infty (-\dot{V}(e,\tilde{p},\tilde{q},\tilde{\phi}))dt$$

$$\int_0^\infty z^T z\, dt < \gamma^2 \int_0^\infty w^T w\, dt$$

$$\Rightarrow \qquad \|z\|_2^2 < \gamma^2 \|w\|_2^2$$

this completes the proof.

Remark 2:

It should be noted that in [23], an H∞ adaptive control (*HAC*) approach is presented for robotic system with disturbance, where tracking was achieved but with slow response. However, in proposed approach, state and reference gains with adaptive error gain are using for fast trajectory tracking performance, which can be seen in example given in the next section.

4. Numerical Simulations

This section presents an example of the proposed H∞ MRAC scheme. A two-link manipulator is selected with parameter uncertainties and external disturbance to validate the efficacy of developed scheme. Moreover, the comparative simulations are presented of the proposed method with HAC scheme.

The structure of the two-link planar robotic manipulator is shown in Fig. 2 [28]. Moreover, the parameter is given in Table 1 and the dynamic equations is presented as follows

$$M(\theta)\ddot{\theta} + H(\theta,\dot{\theta})\dot{\theta} = \tau + w$$

$$\ddot{\theta} = M^{-1}(\tau + w) - M^{-1}H(\theta,\dot{\theta})\dot{\theta}$$

$$\Rightarrow \ddot{\theta} = M^{-1}(\tau + w) - Y(\theta,\dot{\theta},\ddot{\theta})\phi$$

where $M(\theta) = \begin{bmatrix} J_1 & m_2 r_2 l_1 \cos(\theta_2-\theta_1) \\ m_2 r_2 l_1 \sin(\theta_2-\theta_1) & J_2 \end{bmatrix}$,

$$H(\theta) = m_2 r_2 l_1 \sin(\theta_2-\theta_1)\begin{bmatrix} 0 & -\dot{\theta}_2 \\ \dot{\theta}_1 & 0 \end{bmatrix}$$

$$Y(\theta,\dot{\theta},\ddot{\theta}) = \begin{bmatrix} \sin(\theta_2-\theta_1)\dot{\theta}_1^2 & 0 & -\sin(\theta_2-\theta_1)\cos(\theta_2-\theta_1)\dot{\theta}_2^2 & 0 \\ 0 & \sin(\theta_2-\theta_1)\dot{\theta}_2^2 & 0 & -\sin^2(\theta_2-\theta_1)\dot{\theta}_1^2 \end{bmatrix}$$

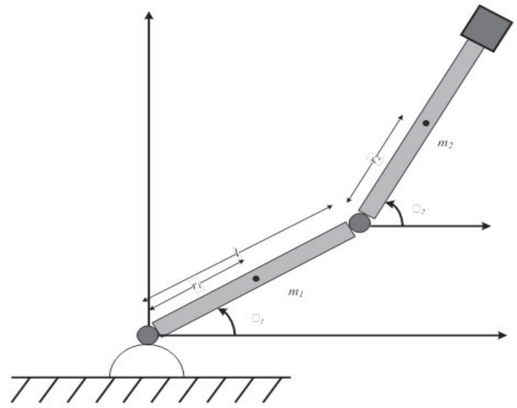

Fig. 2: Two-link robotic manipulator

$$J_1 = \frac{4}{3}m_1 r_1^2 + m_1 l_1^2,\ J_2 = \frac{4}{3}m_2 r_2^2,\ \phi = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \end{bmatrix}^T$$

$$p_1 = m_2 r_2 l_1 J_2,\ p_2 = m_2 r_2 l_1 J_1,\ p_3 = m_2^2 r_2^2 l_1^2,\ p_4 = m_2^2 r_2^2 l_1^2\,.$$

Table 1: Parameters of two-link manipulator

	Link-1	***Link-2***
m (kg)	1	1
r (m)	1	1
l (m)	2	-

Let γ=*0.1* and then applying LMI on (17), then the matrices K, M^{-1} and P are calculated as follows

$$K=\begin{bmatrix}-0.7444 & 0 & 0.6338 & 0\\ 0 & -0.7817 & 0 & 0.9497\end{bmatrix},$$

$$M^{-1}=\begin{bmatrix}1.0527 & 0\\ 0 & 1.1137\end{bmatrix},$$

$$P=\begin{bmatrix}0.0184 & 0 & 0.0042 & 0\\ 0 & 0.0301 & 0 & 0.0041\\ 0.0042 & 0 & 0.0260 & 0\\ 0 & 0.0041 & 0 & 0.0324\end{bmatrix}$$

Moreover, the parameters of the proposed method with appropriate dimensions are given as

$$\Lambda_p=350\times\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 700 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 2\end{bmatrix},\quad \Lambda_q=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad \Lambda_\phi=[1],$$

$$A_r=\begin{bmatrix}-1 & 0 & 0 & 0\\ 0 & -2 & 0 & 0\\ 0 & 0 & -3 & 0\\ 0 & 0 & 0 & -4\end{bmatrix},C=\begin{bmatrix}0.1 & 0 & 0 & 0\\ 0 & 0.1 & 0 & 0\end{bmatrix},$$

$$D=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},\ E=\begin{bmatrix}0.001 & 0\\ 0 & 0.001\end{bmatrix},\quad \theta(0)=\begin{bmatrix}0.4 & 0.4\end{bmatrix}^T,$$

$$\hat{p}(0)=\begin{bmatrix}400 & 600 & 0.001 & 400\\ 1000 & 180 & 600 & 600\end{bmatrix}^T,\quad \hat{q}(0)=\begin{bmatrix}1 & 1\\ 1.35 & 1.4\end{bmatrix}^T,$$

$\hat{\phi}(0)=\begin{bmatrix}2.66 & 10.66 & 4 & 4\end{bmatrix}^T$, and the unit step is chosen as desired input. The comparative results are shown between the proposed method and HAC, the simulations are tested in the absence of external disturbance and the parameters are fairly selected. Therefore, the comparative simulation results of joint position and its error are shown in Fig. 3 and Fig. 4, respectively.

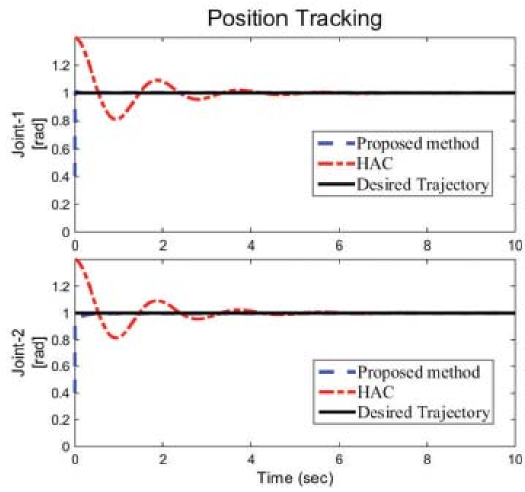

Fig. 3: Position tracking when w=0

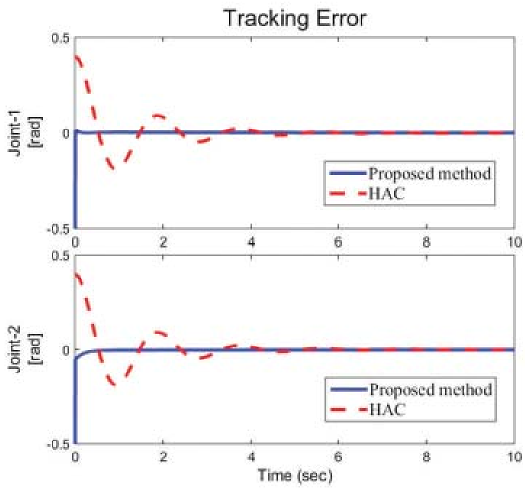

Fig. 4: Tracking error when w=0

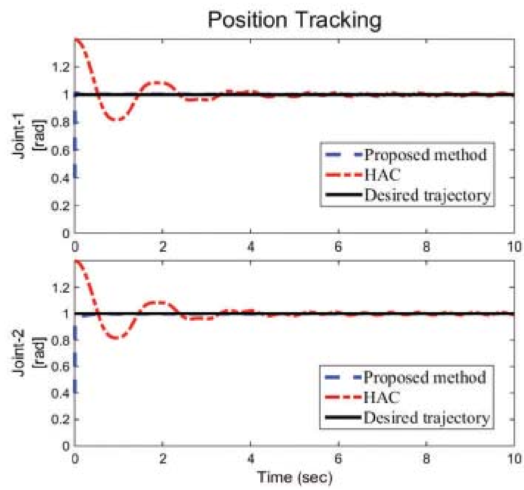

Fig. 5: Position tracking when w≠0

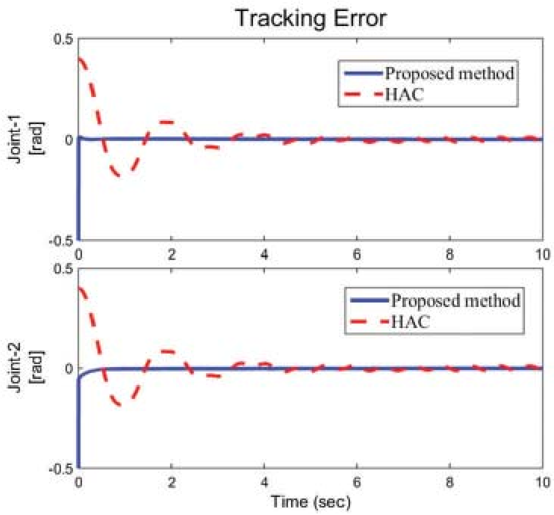

Fig. 6: Tracking error when w≠0

method with external disturbances, and the performance shows that the proposed method obtains the fast convergence, robust position tracking and small error, so that the steady state error achieves at 0.1 sec.

5. Conclusions

The developed scheme is utilized to guarantee that the joints accurately achieve the trajectory tracking for two-link manipulator system. The robust stability and the validation of the robotic manipulator under unknown external disturbance and parameter uncertainty have been presented. A sufficient condition for the robotic manipulator is achieved from H∞ performance analysis. An adaptive control scheme is formulated such that the resultant closed-loop system has a prescribed H∞ disturbance attenuation level γ.

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